The effect of surface and Coulomb interaction on the liquid-gas phase transition of finite nuclei

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Abstract

By means of the Furnstahl, Serot and Tang's model, the effects of surface tension and Coulomb interaction on the liquid-gas phase transition for finite nuclei are investigated. A limit pressure $p_{\rm lim}$ above which the liquid-gas phase transition cannot take place has been found. It is found that comparing to the Coulomb interaction, the contribution of surface tension is dominate in low temperature regions. The binodal surface is also addressed.

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Since the arguments given by Müller and Serot [1] that a second order liquid-gas (L-G) phase transition will take place in a multi-components and multi-conserved charged system, much theoretical efford has been devoted for studying this problem by using different models and different treatments [2-4]. But all investigations are limited to infinite nuclear matter. It is of interest to extend this study to finite nuclei. This is the objective of this paper.

If we consider the finite nuclei as a liquid droplet and discuss its L-G phase transition, two major effects, namely, the surface energy of the droplet and the Coulomb interaction of proton-proton must be considered. The reasons are as follows: it has been shown that the difference of the chemical potentials between proton and neutron plays an essential role to determine the order of L-G phase transition [1, 2]. For the infinite nuclear matter the chemical potentials of proton and neutron depend on the third component I_3 of isospin when nucleon-nucleon-ρ-meson ($NN\rho$) interaction exists. In symmetric nuclear matter, the L-G phase transition is of first order because $I_3 = 0$. In asymmetric nuclear matter, $I_3 \neq 0$ and then the chemical potential of neutron μ_n does not equal to the chemical potential of proton μ_p, a second order phase transition may take place. The Coulomb interaction cannot be taken into account because it becomes divergent in *infinite* nuclear matter. But in *finite* nuclei, the contribution of Coulomb interaction can be considered. Obviously, the chemical potential of proton μ_p not only depends on I_3, but also on Coulomb interaction. But the later has no effect on μ_n. The contribution of Coulomb interaction will make that the values of μ_p and μ_n becomes more different.

Besides Coulomb interaction, on the other hand, the surface tension of the droplet will affect on the pressure of the liquid phase and then on the coexistence equations because the pressures of two phases must equal at the phase transition point.

To exhibit the effects of surface energy and Coulomb interaction on the L-G phase transition for finite nuclei, we employ the Furnstahl-Serot-Tang (FST) model [5-8], which has been shown to be successful to explain the properties of both infinite nuclear matter and finite nuclei. The Lagrangian density of FST model under mean field approximations reads

$$\begin{aligned} L_{MFT} = & \overline{\Psi}\left[i\gamma^{\mu}\partial_{\mu} - (M - g_s\phi_0) - g_v\gamma^0 V_0 - \frac{1}{2}g_\rho\tau_3\gamma^0 b_0\right]\Psi \\ & + \frac{1}{2}m_v^2 V_0^2\left(1 + \eta\frac{\phi_0}{S_0}\right) + \frac{1}{4!}\zeta\left(g_v V_0\right)^4 + \frac{1}{2}m_\rho^2 b_0^2 \\ & - H_q\left(1 - \frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}ln\left(1 - \frac{\phi_0}{S_0}\right) - \frac{1}{4}\right] \end{aligned} \tag{1}$$

where g_s, g_v g_ρ are, respectively, the couplings of light scalar meson σ, vector meson ω and isovector meson ρ fields to the nucleon, ϕ_0, V_0, b_0 are the expectation values $\phi_0 \equiv <\phi>$, $<V_\mu> \equiv \delta_{\mu 0}V_0$, $<b_{\mu 3}> \equiv \delta_{\mu 0}b_0$. The scalar fluctuation field ϕ is related to S by $S(x) = S_0 - \phi(x)$ and H_q is given by $m_s^2 = 4H_q/\left(d^2 S_0^2\right)$, d the scalar dimension. By using the standard technique of statistical mechanics, we get the thermodynamic potential Ω as [2, 9]

$$\begin{aligned} \Omega = V\Bigg\{ & H_q\left[\left(1 - \frac{\phi_0}{S_0}\right)^{4/d}\left(\frac{1}{d}ln\left(1 - \frac{\phi_0}{S_0}\right) - \frac{1}{4}\right) + \frac{1}{4}\right] \\ & - \frac{1}{2}m_\rho^2 b_0^2 - \frac{1}{2}\left(1 + \eta\frac{\phi_0}{S_0}\right)m_v^2 V_0^2 - \frac{1}{4!}\zeta\left(g_v V_0\right)^4\Bigg\} \end{aligned} \tag{2}$$

$$-2k_BT\left[\sum_{k,\tau} ln\left(1+e^{-\beta(E^*(k)-\nu_\tau)}\right)+\sum_{k,\tau} ln\left(1+e^{-\beta(E^*(k)+\nu_\tau)}\right)\right]$$

where $\beta = 1/k_BT$ and the quantity ν_i $(i = n, p)$ is related to the usual chemical potential μ_i by the equations

$$\nu_n = \mu_n - g_v V_0 + \frac{g_\rho^2 \rho_3}{4m_\rho^2} \tag{3}$$

$$\nu_p = \mu_p - g_v V_0 - \frac{g_\rho^2 \rho_3}{4m_\rho^2} \tag{4}$$

where $\rho_3 = \rho_p - \rho_n$, the third component of isospin $I_3 = (N_p - N_n)/2 = V\rho_3/2$, and $E^*(k) = \sqrt{M^{*2} + k^2}$ with $M^* = M - g_s\phi_0$. Usually, instead of ρ_3, we introduce $\alpha = (\rho_n - \rho_p)/\rho$, the asymmetric parameter to calculate, where $\rho = \rho_n + \rho_p$. Having obtained the thermodynamic potential, all other thermodynamic quantities, for example, pressure p, can be calculated. We get

$$\begin{aligned} p = &-H_q\left\{\left(1-\frac{\phi_0}{S_0}\right)^{\frac{4}{d}}\left[\frac{1}{d}ln\left(1-\frac{\phi_0}{S_0}\right)-\frac{1}{4}\right]+\frac{1}{4}\right\} \\ &+\frac{1}{2}\left(1+\eta\frac{\phi_0}{S_0}\right)m_v^2V_0^2+\frac{1}{4!}\zeta g_v^4V_0^4+\frac{g_\rho^2}{8m_\rho^2}\rho_3^2 \\ &+\frac{1}{3}\frac{2}{(2\pi)^3}\int d^3k\frac{k^2}{E^*(k)}[n_n(k)+\overline{n}_n(k)+n_p(k)+\overline{n}_p(k)]. \end{aligned} \tag{5}$$

where

$$n_\tau(k) = \{exp[(E^*(k) - \nu_\tau)/k_BT] + 1\}^{-1} \tag{6}$$

$$\overline{n}_\tau(k) = \{exp[(E^*(k) + \nu_\tau)/k_BT] + 1\}^{-1} \tag{7}$$

$$(\tau = n, p)$$

are the nucleon and anti-nucleon distributions respectively. The neutron density ρ_n and the proton density ρ_p are given by

$$\rho_\tau = \frac{2}{(2\pi)^3}\int d^3k[n_\tau(k) - \overline{n}_\tau(k)]. \qquad (\tau = n, p) \tag{8}$$

Now we are in a position to study the L-G phase transition of finite nuclei. The two-phase coexistence equations are

$$\mu_n'(T, \rho', \alpha') = \mu_n''(T, \rho'', \alpha'') \tag{9}$$

$$\mu_p'(T, \rho', \alpha') + \mu_{Coul}(\rho') = \mu_p''(T, \rho'', \alpha'') \tag{10}$$

$$p'(T, \rho', \alpha') + p'_{Coul}(\rho') + p'_{surf}(T, \rho') = p''(T, \rho'', \alpha'') \tag{11}$$

where the prime and the double prime refer to the liquid phase and gas phase, respectively. Considering the droplet as an uniformly charged sphere, the contribution of Coulomb interaction to the chemical potential of proton and the pressure are [9, 10, 11].

$$\mu_{Coul} = \frac{6}{5}\frac{Ze^2}{R} \tag{12}$$

$$p_{Coul}(\rho) = \frac{Z^2e^2}{5AR}\rho \tag{13}$$

respectively. The additional pressure provided by the surface tension of the liquid droplet is [12, 13]

$$p_{surf}(T,\rho) = -2\gamma(T)/R \tag{14}$$

where

$$\gamma(T) = \left(1.14\mathbf{MeV}fm^{-2}\right)\left[1+\frac{3T}{2T_c}\right]\left[1-\frac{T}{T_c}\right]^{3/2} \tag{15}$$

with T_c being the critical temperature of L-G phase transition in symmetric nuclear matter. We take the liquid droplet along the β -stability line, it satisfies

$$Z = 0.5A - 0.3\times10^{-2}A^{5/3} \tag{16}$$

The parameters be chosen for our numerical calculations are the set T_1 of FST model

$$\begin{aligned} g_s^2 = 99.3, \quad g_v^2 = 154.5, \quad g_\rho^2 = 70.2 \\ m_s = 509MeV, \quad S_0 = 90.6MeV \\ \zeta = 0.0402, \quad \eta = -0.496, \quad d = 2.70 \end{aligned} \tag{17}$$

Our results are summerized in Fig.1-Fig.6. To make our results more transparent, we neglect the Coulomb interaction and the surface effect by taking $R\to\infty$ in Fig.1 and the μ_n, μ_p isobar vs α reduce to that of infinite asymmetric matter. In this Figure, we fix the temperature $T = 10$ **MeV** and the curves a, b, c, d, e correspond to the pressure 0.06, 0.085, 0.100, 0.164 and 0.200 $\mathbf{MeV}fm^{-3}$ respectively. We see that the curves for lower pressures are more complicate than those of the large pressures. When $p = 0.200$ $\mathbf{MeV}fm^{-3}$, curve e has one branch only, but when $p = 0.06$ $\mathbf{MeV}fm^{-3}$, curve a has three branchs. The chemical potentials isobar μ_n'', μ_p'' vs α'' given by the right hand side of eq.(9-11) for the gas phase is shown in Fig.2 where $T = 5$ **MeV** and $p = 0.016$ $\mathbf{MeV}fm^{-3}$. In fact, these curves are the same as that of infinite nuclear matter because the chemical potential and the pressure for the gas phase do not depend on the Coulomb interaction and the surface term. We see in this case both $\mu_p''(\alpha'')$ and $\mu_n''(\alpha'')$ curves have three branchs. The chemical isobar as a function of α for the liquid phase and the gas phase are shown in Fig.3 by solid line and dashed line respectively where we fixed $T = 5$ **MeV** and $p = 0.016$ $\mathbf{MeV}fm^{-3}$. The dashed lines in Fig.3 for gas phase are in fact the same curves as those of Fig.2 except that the range of the α-axis is (0.0, 0.5) instead of (0.0, 1.0). The rectangle construction[1, 2] which represents for the Gibbs' conditions of eq.(9 - 11) for the two-phase equilibrium is

also plotted in Fig.3 by the edges of a rectangle. Due to the effect of Coulomb interaction and the surface energy, the chemical potential isobars for the gas phase and for the liquid phase are very different. We see from Fig.3 that the $\mu_p''(\alpha'')$ and $\mu_n''(\alpha'')$ curves for the gas phase have two branchs shown by dashed lines in the regions $0 < \alpha < 0.5$ respectively, but $\mu_p'(a')$ $\mu_n'(\alpha')$ for the liquid phase has one branch in this region only. This behavior is quite different from that of infinite nuclear matter in which the liquid phase and the gas phase chemical isobars μ_n and μ_p are shown by the same curves (for example , see curve a of Fig.1). This new feature leads to multi-solutions for Gibbs' condition because one can find four rectangles between two branchs of $\mu_n''\left(\mu_p''\right)$ and one branch of $\mu_n'\left(\mu_p'\right)$. But according to the equilibrium condition: the chemical potential of the system in equilibrium state must take the minimal value at fixed temperature and pressure [14]. Therefore, only one rectangle which corresponds to the minimum chemical potential isobar shown by solid lines in Fig.3 refers to a stable equilibrium phase transition, and the others are all metastable states. The other three rectangles are not shown in Fig.3.

The section of binodal surface [1,2] at finite temperature $T = 5$ **MeV** is shown in Fig.4. A limit pressure $p_{\lim} = 0.018\mathbf{MeV}fm^{-3}$ above which the rectangle cannot be found and the coexistenced equations eq.(9-11) have no solution has been obtianed. The binodal surface will cut off at limit pressure $p_{\lim}$. This situation is very similar to that of our previous paper [2] in which we considered the density dependence of the $NN\rho$ coupling $g_\rho(\rho)$. The reason is that no matter $g_\rho(\rho)$ or the Coulomb interaction or the surface energy, even though they change the chemical potential μ_n and μ_p in different fashions, they will make that the rectangle construction turns out to be disappear. In fact, this result is a reflaction of the so called Coulomb instability in finite nuclei [9-11, 13, 15]. The Coulomb instability of FST model has been discussed in detail by our previous paper[9] in which we found a limit temperature $T_{\lim}$ above which the coexistenced equations have no solution and the L-G phase transition can not take place. The difference is that, instead of $T_{\lim}$, we now fix temperature $T = 5$ **MeV** to find the limit pressure.

Finally, we hope to compare the effects of Coulomb interaction and the surface tension on the L-G phase transition, separably. As shown in eq.(13) and eq.(14), we see that, firstly, the effects of Coulomb interaction and the surface tension are opposite because of p_{Coul} and p_{surf} with opppsite signs, and secondly, p_{surf} depends on temperature but p_{Coul} is independent. It means that the Coulomb interaction and the surface tension play different roles in different temperature regions. In low temperature regions $T \leq 5$ **MeV** $\ll T_c$, $\gamma(T)$ becomes larger, and we have $|p_{surf}| > p_{Coul}$, the surface tension becomes dominant. To show this result clearly, we draw the chemical isobar vs. α curves with the surface effect only and with the Coulomb interaction only in Fig.5 and Fig.6 respectively. Comparing Fig.5, Fig.6 and Fig.3 we see that the curves of Fig.5 are very similar to those of Fig.3, but the curves of Fig.6 are very different. It confirms the surface effect dominates at $T = 5\mathbf{MeV}$. The curves in Fig.6 are very complicate. The reason is that we see from eq.(11), in the phase transition process, p'_{surf} will increase p'' since it has negative sign, but p'_{Coul} will decrease p''. As indicated by Fig.1, the pressure p'' becomes lower, the curve becomes more complicate.

In summary, it is shown that the surface effect and the Coulomb interaction are important for the L-G phase transition of finite nuclei. In low temperature $T \ll T_c$ regions, the surface effect is dominate. A limit pressure $p_{\lim}$ above which the L-G phase transition cannot take place has been found. Since the critical temperature $T_c = 14.75$ **MeV**, and the limit

temperature $T_{\rm lim}$ is around $5.4-8.8$ **MeV** for FST model [12], we come to a conclusion that the surface effect is dominate in the L-G phase transition of finite nuclei for FST model.

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FIGURES

FIG. 1. The chemical isobars for infinite nuclear matter, where $T = 10$ **MeV**, and a, b, c, d, e refer to the pressure 0.06, 0.085, 0.100, 0.164 and 0.200$\mathbf{MeV}fm^{-3}$ respectively.

FIG. 2. The chemical isobars for the gas phase where $T = 5$ **MeV** and $p = 0.016$ $\mathbf{MeV}fm^{-3}$.

FIG. 3. The rectangle construction for two-phase equilibrium for $T = 5$ **MeV** and $p = 0.016$ $\mathbf{MeV}fm^{-3}$.

FIG. 4. The section of binodal surface for $T = 5$ **MeV**.

FIG. 5. The chemical isobars vs. α curves with surface effect only.

FIG. 6. The chemical isobars vs. α curves with Coulomb interaction only.